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STEADY COOLING EFFECT OF AN ANISOTROPIC THERMOELECTRIC REFRIGERATOR

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The theory of cooling of an anisotropic thermoelectric refrigerator (ATR) is given on the assumption that the temperature is two-dimensional along the longitudinal section.

A one-dimensional stationary model of ATR cooling (the temperature is one-dimensional over the cross section of the ATR specimen) was used as early as in [1] for calculating the operating characteristics of thermoelectric devices of transverse type. It was also used in [2]. A one-dimensional model mainly adequately describes the essence of the thermal processes that occur in an ATR; however, it gives no way of taking into account the heat transfer of the end and lateral faces with the surroundings. A two-dimensional model for studying the influence of the heat transfer of the lateral faces with the surroundings on the cooling effect is described in [3], where it is shown that, for an infinitely long specimen, the temperature distribution over the cross section of the refrigerator in the general case is two-dimensional. Two-dimensionality can be disregarded on condition that the aspect ratio of the specimen is much smaller than $\pi/4$ and, if the lateral faces are adiabatically insulated from the surroundings, also at $\alpha_{31} = 0$ for any aspect ratio. It should be noted that the model proposed in [3] neglects the heat transfer of the ATR ends with the surroundings.

At the same time, it is clear that thermal contact of the ends with the surroundings leads to a heat inflow into the specimen and, correspondingly, to a weakening of the cooling effect.

Thermal conditions can be selected differently. However, we think that isothermal contact is the closest to the actual experimental situation.

Indeed, the input leads to the specimen are made of a metal with a high electrical conductivity (for example, copper). The cross section of the input leads is chosen the same as that of the specimen – this facilitates the generation of a one-dimensional electric current in the specimen that is close to a direct one. The high electrical conductivity of the metal of the input leads also implies its high thermal conductivity; therefore the experimental situation is similar to that presented in Fig. 1.

Let us find a stationary temperature distribution in the ATR specimen, assuming that the material of which it is made is thermoelectrically anisotropic, i.e., anisotropic only with respect to thermal emf, and the kinetic coefficients are independent of the temperature and coordinates. On condition that the temperature is a function of x and y (see the figure) and the current traverses the specimen along the axis, a stationary temperature distribution is found from the generalized heat-conduction equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \gamma = 0 . \quad (1)$$

In the general case, the temperature must also be a function of the coordinate z. However, given that $\alpha_{31} = 0$ (i.e., the crystallographic axis coincides with the z axis) and the lateral faces of the specimen are adiabatically insulated from the surroundings, this relation can be disregarded [3].

The boundary conditions

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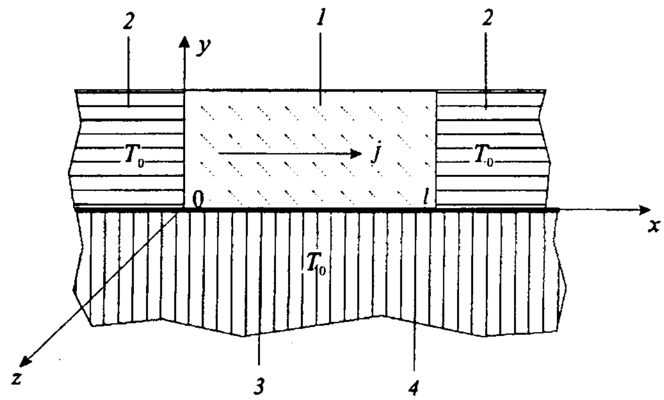

Fig. 1. Longitudinal section of the ATR by the $x0y$ plane: 1) ATR specimen; 2) input leads to the specimen; 3) spacer made of dielectric with a high thermal conductivity, for example, beryllium ceramics; 4) thermostat.

$$T(0, y) = T(l, y) = T(x, 0) = T_0 , \tag{2}$$

$$\frac{\partial T(x, h)}{\partial y} - aT(x, h) = 0 \tag{3}$$

imply the following: (2) isothermal contact of specimen 1 (see the figure) with thermostat 4 at the temperature T_0 of its bottom face ($y = 0$) and ends ($x = 0$, $x = l$) and (3) adiabatic insulation of the top face. It should be noted that isothermal contact of the bottom face of the specimen with a thermostat provides the removal of a part of the released Joule heat (the other part is balanced out by the transverse Peltier effect) and, if any, of the heats of a different nature – this is the principle of operation of an ATR. As for the thermostatic control of the end faces, it can only weaken the cooling effect.

Let the solution of Eq. (1) be represented as

$$T(x, y) = T_0 - \frac{1}{2}\gamma y^2 - \frac{\gamma h}{2}\frac{2 + ah}{1 + ah} y - \frac{aT_0}{1 + ah} y + \sum_{n=1}^{\infty} f_n(x) \sin\left(\frac{\xi_n}{h} y\right). \tag{4}$$

Expression (4) satisfies the condition $T(x, 0) = T_0$, and the condition of adiabatic insulation (3) leads to the transcendental equation

$$\tan \xi_n = \frac{1}{ah} \xi_n , \tag{5}$$

whose solutions are known [4].

Equation (1) and expression (4) yield

$$f_n(x) = A_n \exp\left(\frac{\xi_n}{h} x\right) + B_n \exp\left(-\frac{\xi_n}{h} x\right), \tag{6}$$

where A_n and B_n are constants, which are determined with the aid of boundary conditions (2):

$$A_n = \frac{-\frac{1}{2}\gamma C_n + \frac{\gamma h}{2}\frac{2 + ah}{1 + ah} D_n}{2 \sinh\left(\frac{\xi_n l}{h}\right)} \left[1 - \exp\left(-\frac{\xi_n l}{h}\right)\right],$$

$$B_n = \frac{-\frac{1}{2}\gamma C_n + \frac{\gamma h}{2}\frac{2+ah}{1+ah}D_n}{2\sinh\left(\frac{\xi_n l}{h}\right)}\left[1-\exp\left(\frac{\xi_n l}{h}\right)\right],$$

where

$$C_n = \frac{2}{h}\int_0^h y^2 \sin\left(\frac{\xi_n}{h}y\right)dy\,,\quad D_n = \frac{2}{h}\int_0^h y \sin\left(\frac{\xi_n}{h}y\right)dy$$

are the factors of the Fourier expansion in terms of sines of y^2 and y, respectively.

Simple calculations lead to the following expressions for D_n and C_n:

$$D_n = \frac{2h}{\xi_n}\left(-\cos\xi_n + \frac{\sin\xi_n}{\xi_n}\right),\quad C_n = \frac{2h^2}{\xi_n}\left[-\cos\xi_n + \frac{2}{\xi_n}\sin\xi_n + \frac{1}{\xi_n}(\cos\xi_n - 1)\right].$$

On substituting A_n and B_n into expressions (6) and (4), we obtain the expression for the temperature distribution in the ATR specimen:

$$T(x, y) = T_0 - \frac{1}{2}\gamma y^2 + \frac{\gamma h}{2}\frac{2+ah}{1+ah}y - \frac{aT_0}{1+ah}y -$$

$$-\sum_{n=1}^{\infty}\frac{-\frac{1}{2}\gamma C_n + \left(\frac{\gamma h}{2}\frac{2+ah}{1+ah} - \frac{aT_0}{1+ah}\right)D_n}{\sinh\left(\frac{\xi_n l}{h}\right)}\left[\sinh\left(\frac{\xi_n}{h}(l-x)\right) + \sinh\left(\frac{\xi_n}{h}x\right)\right]\sin\left(\frac{\xi_n}{h}y\right).$$

As is seen from the resulting expression, the temperature distribution along the longitudinal section of the ATR specimen is two-dimensional. To evaluate the cooling effect numerically, we select points that are equidistant from the specimen ends, i.e., set $x = l/2$. (It should be noted that the point $(l/2, h)$ is selected in experimental investigations.) Then, the l-dependent factor under the summation sign is simple in form: $\cosh^{-1}(\xi_n l/2h)$. On condition that $l \to \infty$, the temperature in the middle of the specimen is a function of only y, and at the point $y = h$ it is as follows:

$$T(h) = \frac{T_0 + \frac{1}{2}\frac{\rho}{\kappa}(jh)^2}{1 + \frac{\alpha_{12}}{\kappa}jh}. \tag{7}$$

Hence, the optimum current density is

$$j_{\text{opt}} = \frac{\sqrt{1 + 2ZT_0} - 1}{\alpha_{12}h}\kappa. \tag{8}$$

Substituting Eq. (8) into Eq. (7) yields the expression for the minimum temperature:

$$T_{\min} = \frac{\sqrt{1 + 2ZT_0} - 1}{Z}. \tag{9}$$

It is valid for any Z. For small Z, when $2ZT_0 \ll 1$, we obtain [1]

$$T_{\min} = T_0 - \frac{1}{2} Z T_0^2 .$$

An expression for the temperature at the point $(l/2, h)$ is of the form

$$T(l/2, h) = T(h) - \sum_{n=1}^{\infty} \frac{-\frac{1}{2}\gamma C_n + \left(\frac{\gamma h}{2}\frac{2+ah}{1+ah} - \frac{aT_0}{1+ah}\right) D_n}{\cosh\left(\frac{\xi_n l}{2h}\right)} \sin \xi_n .$$

The sum in the last expression is always negative. Therefore, the larger in magnitude it is, the weaker the cooling effect. Its magnitude depends on the characteristics of the material of the ATR specimen, the current density, and the dimensions l and h. Let the current of the density j_{opt} traverse the specimen lengthwise; then, at T_0 = 300 K, $Z = 10^{-3}$ K^{-1}, and ah = 0.3 and, according to [4], ξ_1 = 1.40 and ξ_2 = 4.65, for which at h = 0.5 cm and l = 2.5 cm, cosh $(\xi_2 l/2h) \approx 16$ and cosh $(\xi_2 l/2h) \approx 6 \cdot 10^4$. Therefore, the sum can be neglected in comparison with $T(h) = T_{\min}$ = 265 K. Thus, in this case, at $l/h \sim 5$ the heat inflow inside the specimen through the ends does not affect the cooling effect. At other parameters of the specimen material and another magnitude of the current, the aspect ratio is different, specifically, the smaller the Z, the larger the aspect ratio that should be chosen. The experimental value of l/h was 4.9 in [5] and 5 in [6], which is in agreement with theoretical values.

NOTATION

x, y, z, axes of laboratory system of coordinates; T, temperature; T_0, temperature of thermostat; ρ and κ, specific resistance and thermal conductivity; j, current density; h and l, height and length of specimen; α_{12}, α_{31}, components of tensor of thermal emf; $Z = \alpha_{12}^2(\kappa\rho)$, anisotropic thermoelectric factor of merit; n = 1, 2, 3, ..., index of summation; j_{opt}, optimum current density; $T_{\min}$, minimum temperature; $a = \alpha_{12} j/\kappa$, $\gamma = \rho j^2/\kappa$; ξ_n, roots of transcendental equation (5).

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